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Numerical study of the solid particle motion in grid-generated turbulent flows

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Abstract—Applying the basic idea of time series analysis to the fluid Lagrangian temporal correlation and Eulerian spatial correlation functions, a model for particles suspended in turbulent flows has been formulated. This model contains both the time and space effects of turbulence on particles. As a validation of the present model, the experiments of Synder and Lumley and of Wells and Stock are simulated numerically. A comparison of the calculated and experimental results, including the particle-transverse dispersions and fluctuating velocity decays, is made. Besides, particle-dispersions and fluctuating velocity decays in the longitudinal direction are also computed in order to examine the effects of the particle inertia and the crossing trajectories.

1. INTRODUCTION

THE DISPERSION of particles by turbulent flows can be found in many engineering applications, such as pulverized-coal combustors, diesel-engine sprays, aerosols, and rocket plumes. Our particular concern is with ventilation flows. The objective of our research is to provide methods of predicting correctly the distribution of materials harmful to people working inside premises so that we could assess the air quality inside these premises.

During the past few years, many numerical methods have been developed to describe turbulent flows laden with particles. The models are based either on a Eulerian formalism (for example, Abbas *et al.* [1], Durst *et al.* [2], Elghobashi *et al.* [3]) or on a Lagrangian formulation. In the Lagrangian methods, the trajectories of particles transported by the turbulent fluid flow are computed by solving the corresponding equation of motion. The main difficulty lies in the fact that usually only the mean fluid velocity is known and that the fluctuating part of the velocity has to be modeled from turbulent characteristics of the flow. The fluctuating fluid velocity is obtained from a random process. Obviously, the velocities are correlated in space and time and the different existing models differ by the forms adopted for the Lagrangian autocorrelation function and the Eulerian spatial correlation function. They also differ by the method of generating the random processes. One of the simplest Lagrangian models has been proposed by Gosman and Ioannides [4] who assume very simple correlation functions. An improvement of this model has been presented by Ormancey and Martinon [5]. In their model the Lagrangian autocorrelation decreases as an exponential function. They also allow for two Eulerian spatial correlations; one is longitudinal and the other is transverse. Berlemont *et al.* [6] pursued this work and could adopt a more general form for the fluid Lagrangian correlation function. Burnage and Moon [7] presented another improvement of the work of Gosman and Ioannides based on two Poisson processes; one is for space correlations and the other is for time correlations.

In this paper, we propose a new Lagrangian model based on the idea of time series analysis. Two successive fluid velocities which differ by one time step are related to each other by a Markov process, as done by Parthasarathy and Faeth [8]. The new idea inherent in this paper is to relate also the two velocities computed at two different points in space (close to each other) by a Markov process. Combining both Markov processes, we are then able to compute the fluctuating fluid velocities at the successive points of the particle trajectory. This method is quite simple and does not necessitate following a fluid particle during several time steps, as done by previous authors. We change the fluid particle every time step. This paper only describes an application of our Lagrangian model to the simulation of the experiments of Snyder and Lumley [9] and of Wells and Stock [10] for the particle-dispersions and velocity decays. Moreover, some additional quantities are also computed so as to provide insight into the crossing-trajectory effect [11].

2. ESTABLISHMENT OF THE METHOD

2.1. *Particle motion*

When neglecting the influence of streamline curvature and the interaction between particles, the motion of a spherical and rigid particle in a fluid flow is governed by the following equations [12]:

NOMENCLATURE

- a_i coefficients in equation (10)
- A_c coefficient in equation (1)
- b_i coefficients in equation (10)
- C_A, C_D, C_H coefficients for added mass, drag and history terms in equation (1)
- d_p particle diameter
- D coefficient in equation (16)
- exp exponential function
- f_{ii} Lagrangian autocorrelation function
- g acceleration due to gravity
- g_{ii} Eulerian spatial correlation function
- k turbulent fluctuating kinetic energy
- M grid spacing
- $(Re)_p$ particle Reynolds number
- Δs distance between particle and fluid in one time step of calculation
- t time
- Δt time step of calculation
- u fluid fluctuating velocity
- $\overline{u_i^2}$ fluid fluctuating velocity variance
- $\mathbf{U}$ fluid instantaneous velocity vector
- $\mathbf{V}$ particle instantaneous velocity vector
- V_d particle terminal (drift) velocity
- X absolute space coordinate
- $\mathbf{X}$ position vector in the absolute coordinate system O–XY
- Y absolute space coordinate
- Z normalized fluid fluctuating velocity.

Greek symbols

- α mean-zero random variable
- β mean-zero random variable
- γ mean-zero, Gaussian random variable
- ε turbulent kinetic energy dissipation rate
- ζ relative space coordinate
- η relative space coordinate
- θ angle formed by the ζ and X axes
- Λ lengthscale
- ν kinematic viscosity coefficient of fluid
- ρ density
- σ_γ standard deviation of γ
- τ timescale.

Subscripts

- 1 η direction
- 2 ζ direction
- f fluid point
- p particle
- s starting point of each time step
- X X direction
- Y Y direction.

$$\rho_p \frac{d\mathbf{V}}{dt} = -\frac{3}{4d_p}\rho_f C_D(\mathbf{V}-\mathbf{U})|\mathbf{U}-\mathbf{V}| - \rho_f C_A \frac{d(\mathbf{V}-\mathbf{U})}{dt} + (\rho_p-\rho_f)g + \rho_f \frac{D\mathbf{U}}{Dt} - \frac{3\rho_f C_H}{2d_p}\sqrt{\left(\frac{\nu}{\pi}\right)}\int_{-\infty}^{t}\frac{d(\mathbf{V}-\mathbf{U})}{dt}\frac{1}{\sqrt{(t-\tau)}}d\tau \quad (1)$$

$$\frac{d\mathbf{X}}{dt} = \mathbf{V} \quad (2)$$

where d/dt and D/Dt are the temporal derivatives along the particle and fluid trajectories, respectively. C_D, C_H and C_A are coefficients introduced by Riley which depend on the particle volume and particle Reynolds number $(Re)_p$. They are given by [13, 14]

$$C_D = \frac{24}{(Re)_p}(1+0.15(Re)_p^{0.687}) \quad \text{for } (Re)_p < 200$$

and for $(Re)_p < 60$

$$C_A = 1.05 - \frac{0.0666}{A_c^2+0.12}$$

$$C_H = 2.88 + \frac{3.12}{(A_c+0.12)^3}$$

where

$$A_c = \frac{|\mathbf{U}-\mathbf{V}|^2}{d_p\left|\frac{d(\mathbf{V}-\mathbf{U})}{dt}\right|}$$

and $(Re)_p$ is defined by

$$(Re)_p = \frac{|\mathbf{U}-\mathbf{V}|\,d_p}{\nu}.$$

In the present study, the Basset term, the temporal derivatives d/dt and D/Dt of the fluid fluctuating velocity, are neglected. Such a simplification was justified for low turbulence intensities and moderate departure from homogeneity [15]. The corresponding derivatives of the fluid mean velocity are preserved. The mean fluid velocity of the turbulent flows to be discussed is uniform; in such cases, equation (1) reduces to

$$\rho_p \frac{d\mathbf{V}}{dt} = -\frac{3}{4d_p}\rho_f C_D(\mathbf{V}-\mathbf{U})|\mathbf{U}-\mathbf{V}| - \rho_f C_A \frac{d\mathbf{V}}{dt} + (\rho_p-\rho_f)g.$$

In the two experiments considered, the particle mass-loading is so low that the presence of particles does not modify the fluid motion. Therefore all the mean data of the fluid turbulent field, including the mean velocity, the turbulent kinetic energy and the turbulent kinetic energy dissipation rate, are regarded as known *a priori*. To know the statistic properties of particles, we have to follow each particle along its trajectory. In order to be able to integrate equations (1) and (2), it is necessary to know the instantaneous velocity of the fluid at the location points of the particle. Since the fluid mean velocity is assumed to be known, the fluctuating velocity of the fluid has to be estimated at the location points of the particle.

2.2. *Modeling of the fluctuating velocity of the fluid at the location points of the particle*

The present discussion is limited to two-dimensional situations. We begin to model the fluid fluctuating velocity at the location points of the particle. At time t, the particle and a corresponding fluid point start out from the same position $\mathbf{X}_s$. After Δt, they arrive at $\mathbf{X}_p$ and $\mathbf{X}_f$, respectively, and the distance between $\mathbf{X}_p$ and $\mathbf{X}_f$ is Δs, as shown in Fig. 1. Here $\mathbf{X}_s$, $\mathbf{X}_f$ and $\mathbf{X}_p$ are the position vectors in the absolute coordinate system O–XY. The relative coordinate system o–$\zeta\eta$ is chosen such that its original point, o, is sited at the position $\mathbf{X}_f$ and the η axis passes through the position $\mathbf{X}_p$. Since, in practical applications, the flow fields are often inhomogeneous and non-stationary, to lessen the effects of the reference time and position, we normalize the fluid fluctuating velocity u_i by the square root of its local variance and denote the normalized fluctuating component in the i direction by Z_i, i.e.

$$Z_i = u_i/\sqrt{(\overline{u_i^2})} \quad (i = 1,2)$$

where the subscripts 1 and 2 denote respectively the η and ζ directions in the relative coordinate system o–$\zeta\eta$ shown in Fig. 1 and the overbar indicates the ensemble average values.

In this paper, we shall assume turbulence to be isotropic, which means that the Reynolds stress components $\overline{u_i u_j}$ are zero for $i \neq j$. We assume that the normalized fluctuating velocities at positions $\mathbf{X}_s$, $\mathbf{X}_f$ and $\mathbf{X}_p$ have the following correlation relations:

$$\overline{Z_i(\mathbf{X}_f)Z_i(\mathbf{X}_s)} = \overline{Z_i(\mathbf{X}_s)Z_i(\mathbf{X}_s)}f_{ii}(\Delta t) \quad (i = 1,2) \quad (3)$$

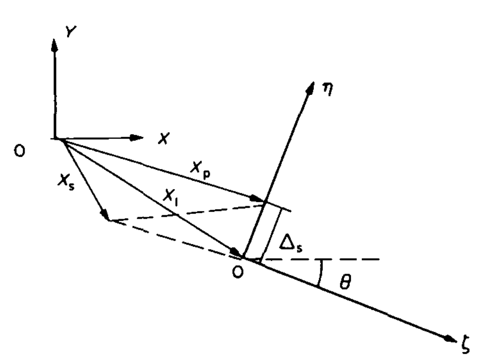

FIG. 1. The locations of the particle and the fluid point at the instants t and $t+\Delta t$.

$$\overline{Z_i(\mathbf{X}_p)Z_i(\mathbf{X}_f)} = \overline{Z_i(\mathbf{X}_f)Z_i(\mathbf{X}_f)}g_{ii}(\Delta s) \quad (i = 1,2) \quad (4)$$

(throughout this paper, no summation convention is used). Relations (3) and (4) are called, respectively, the Lagrangian autocorrelation and Eulerian spatial correlation functions.

In the time series analysis [16], a linear combination of the previous values of some random variable and white noise is constructed in order to foresee the next value of this random variable. The simplest way to apply this idea to the stochastic process associated with the fluid fluctuating velocity is expressed in equation (5) below. Actually, equation (5) had already been used by Durbin [17] to deal with fluid dispersion

$$Z_i(\mathbf{X}_f) = a_i Z_i(\mathbf{X}_s) + \alpha_i \quad (i = 1,2) \quad (5)$$

where a_i $(i = 1,2)$ are coefficients to be determined. Here α_i $(i = 1,2)$, unlike the conventional method, are only mean-zero, random variables independent of $Z_i(\mathbf{X}_s)$ but not necessarily Gaussian. The properties of α_i will be discussed later. Multiplying both sides of equation (5) by $Z_i(\mathbf{X}_s)$ and then taking ensemble average (α_i and $Z_i(\mathbf{X}_s)$ are independent), we obtain

$$\overline{Z_i(\mathbf{X}_f)Z_i(\mathbf{X}_s)} = a_i \overline{Z_i(\mathbf{X}_s)Z_i(\mathbf{X}_s)}. \quad (6)$$

From equations (3) and (6), we are able to determine the coefficient a_i

$$a_i = f_{ii}(\Delta t). \quad (7)$$

The new idea of this paper is to apply the same method to relate two fluid fluctuating velocities estimated at the same time but at two spatial locations $\mathbf{X}_f$ and $\mathbf{X}_p$ separated by the space distance Δs. We therefore assume the following equation

$$Z_i(\mathbf{X}_p) = b_i Z_i(\mathbf{X}_f) + \beta_i \quad (i = 1,2) \quad (8)$$

where b_i $(i = 1,2)$ and β_i $(i = 1,2)$ have the same sense as a_i and α_i in equation (5). By the same procedure adopted for equation (5), from equation (8), we have

$$b_i = g_{ii}(\Delta s). \quad (9)$$

Introducing equation (5) into equation (8) to eliminate $Z_i(\mathbf{X}_f)$, we have

$$Z_i(\mathbf{X}_p) = a_i b_i Z_i(\mathbf{X}_s) + \gamma_i \quad (i = 1,2) \quad (10)$$

where γ_i, being $b_i\alpha_i + \beta_i$, is assumed to be mean-zero, Gaussian random variables. To complete equation (10), we should know the standard deviations $(\sigma_\gamma)_i$ of γ_i. For this, squaring equation (10) and then taking the ensemble average, we have

$$\overline{Z_i^2(\mathbf{X}_p)} = a_i^2 b_i^2 \overline{Z_i^2(\mathbf{X}_s)} + 2a_i b_i \overline{Z_i(\mathbf{X}_s)\gamma_i} + \overline{\gamma_i^2}.$$

Considering the independence of γ_i and $Z_i(\mathbf{X}_s)$ and using the definitions of Z_i and γ_i, we finally arrive at

$$1 = a_i^2 b_i^2 + (\sigma_\gamma)_i^2$$

i.e.

$$(\sigma_\gamma)_i = \sqrt{(1 - a_i^2 b_i^2)} \quad (i = 1,2). \quad (11)$$

Equation (10) with the standard deviation given by equation (11) will be used as the model equation of the present study. It relates directly the fluctuating velocities of the fluid at the successive location points of the particle and includes both the time and the space effects of the turbulent field via coefficients a_i and b_i. In this model, unlike in conventional time series method, α_i and β_i are not supposed to be mean-zero, Gaussian random variables; we only require that their linear combinations γ_i are mean-zero, Gaussian random variables. This can be regarded as the requirement for the properties of α_i and β_i. It is noted that the coordinate system o–$\zeta\eta$ employed in Fig. 1 is local and should be transformed at each time step.

2.3. *Calculation procedure*

(1) The particle position and velocity are given at $t = 0$. The fluid initial fluctuating velocity components u_i ($i = 1, 2$) at the particle position are obtained from Gaussian variables satisfying p.d.f. with variances $\overline{u_i^2}$. The fluid instantaneous velocity can then be obtained by adding the known mean velocity and the fluctuating part.

(2) From the fluid instantaneous velocity found above, the fluid point position at the instant Δt, $\mathbf{X}_f$, can be calculated by the Euler–Cauchy method. Using the given particle initial velocity and the fluid instantaneous velocity just obtained, through equation (1), we can find the position of the particle at time Δt, $\mathbf{X}_p$, by the Runge–Kutta method so that Δs can be calculated (see Fig. 1).

(3) According to the rule of Fig. 1, we establish a relative coordinate system o–$\zeta\eta$ so that we can determine the coefficients a_i and b_i from equations (7) and (9).

(4) Substituting a_i and b_i into equation (10) and generating the mean-zero Gaussian variables γ_i with standard deviations given by equation (11), the fluctuating velocity of the fluid point at $\mathbf{X}_p$ can then be obtained from equation (10). We can further find the fluid instantaneous velocity.

(5) Letting $\mathbf{X}_p$ computed at step (2) be the starting point of the next time step, i.e. $\mathbf{X}_s$, we repeat above steps after the completion of computation. It should be pointed out that at every time step, a new relative coordinate system o–$\zeta\eta$ is used.

3. SIMULATION AND RESULTS

3.1. *Choice of the correlation functions and the involved parameters*

The experiments of Synder and Lumley, Wells and Stock are two, relatively speaking, comprehensive experiments about particle motion in turbulent flows and are often chosen as a basis for testing computational procedures. We also do so and in the present study the following autocorrelation and spatial correlation functions proposed by Frenkiel [18] are adopted:

$$\overline{Z_i(\mathbf{X}_f)Z_i(\mathbf{X}_s)} = \overline{Z_i(\mathbf{X}_s)Z_i(\mathbf{X}_s)}\exp\left(-\frac{\Delta t}{\tau_i}\right) \quad (i = 1, 2) \tag{12}$$

$$\overline{Z_i(\mathbf{X}_p)Z_i(\mathbf{X}_f)} = \overline{Z_i(\mathbf{X}_f)Z_i(\mathbf{X}_f)}\exp\left(-\frac{\Delta s}{2\Lambda_i}\right)\cos\left(\frac{\Delta s}{2\Lambda_i}\right) \quad (i = 1, 2) \tag{13}$$

where Λ_1 and Λ_2 are called, respectively, the longitudinal and transverse lengthscales. Of course, there are many alternatives to the forms of the correlation functions in equations (12) and (13). Throughout all the computation, the integral timescales τ_i and lengthscales Λ_i in equations (12) and (13) are estimated by

$$\tau_1 = \tau_2 = 0.235\frac{\overline{u^2}}{\varepsilon} \tag{14}$$

$$\Lambda_1 = 2.5\tau_1\sqrt{(\overline{u^2})} \tag{15}$$

and

$$\Lambda_1 = D\Lambda_2 \tag{16}$$

where $\overline{u^2} = \frac{1}{2}(\overline{u_1^2}+\overline{u_2^2})$. Here $D = 2$, a conclusion from the stationary, homogeneous, isotropic and incompressible turbulence theory [19] except otherwise noted. Equations (15) and (16) are known as Hinze's relation and Saffman's relation, respectively [19]. In addition, τ_i and Λ_i are estimated at locations $\mathbf{X}_s$ and $\mathbf{X}_f$ (see Fig. 1), respectively. In isotropic or weak anisotropic turbulence, velocity variances evaluated in the absolute system O–XY and the relative coordinate system o–$\zeta\eta$ shown in Fig. 1 are related through

$$\overline{u_1^2} = \sin^2\theta\overline{u_X^2}+\cos^2\theta\overline{u_Y^2}$$

$$\overline{u_2^2} = \cos^2\theta\overline{u_X^2}+\sin^2\theta\overline{u_Y^2}.$$

3.2. *Simulation of the experiment of Snyder and Lumley* [9]

One of the most comprehensive experiments about particle motion in turbulent flow is that reported by Snyder and Lumley [9]. They used particles of various sizes and densities ranging from light particles that would closely follow the fluid velocity fluctuation to heavy particles that would experience both inertia and crossing-trajectory effects. In this experiment the particles are injected into a grid-generated turbulent air flow at $X/M = 20$. The measurements were carried out beginning from $X/M = 68$, where X represents the distance from the grid and $M = 2.54$ cm. The principal direction of the flow, the X direction say, was ascendant. The average flow field parameters are given by

$$\bar{U}_X = 655\,\mathrm{cm\,s^{-1}} \quad \bar{U}_Y = 0 \tag{17}$$

$$\overline{u_X^2} = \frac{(\overline{U_X})^2}{42.4(X/M-16)} \tag{18}$$

$$\overline{u_Y^2} = \frac{(\overline{U_X})^2}{39.4(X/M-12)}. \tag{19}$$

With Taylor's frozen hypothesis and the relation $dk/dt = -\varepsilon$, where k is defined as

$$k = \tfrac{1}{2}(\overline{u_X^2} + 2\overline{u_Y^2}) \tag{20}$$

we obtain

$$\varepsilon = \frac{(\overline{U}_X)^3}{2M}\left[\frac{1}{42.4(X/M-16)^2} + \frac{2}{39.4(X/M-12)^2}\right]. \tag{21}$$

In the simulation of this experiment, the coefficient D in equation (16) is set equal to 2.56, a measured value given by Snyder and Lumley. In the present study, the time step Δt is equal to 0.002 s.

The calculated and experimental transverse dispersions for four types of particles are compared in Fig. 2, where the computed particle-dispersions in the longitudinal direction, i.e. the X direction, are also included. A fair agreement is observed, even for the hollow glass particle, for which some numerical methods [7, 8] and theoretical models [12, 14] fail. The present predictions are in accordance with that of Walklate [20]. It can be seen that for the hollow glass particle, whose time constant is 1.735 ms, the crossing-trajectory effect still cannot be completely neglected for the statistics of dispersion. As a result, dispersions in the X and Y directions are not identical. Besides, the crossing-trajectory effect reduces unequally the particle-dispersions in the directions normal to and parallel to the gravity direction; particles disperse more in the gravity direction.

Figure 3 shows the numerical and experimental particle transverse fluctuating velocity decay curves for the four types of particles. Also included in Fig. 3 are the predicted particle longitudinal fluctuating velocity decay curves and the turbulent flow decay curves. A qualitative agreement between the predicted and experimental data is observed. As realized by Snyder and Lumley, the hollow glass particle should have closely followed the turbulent flow field (considering the loss of the energy due to the low sampling rate used in their experiment). This speculation is confirmed by the present calculation. It is noted that the hollow glass particle indeed responds to all the fluctuations in the turbulent flow and consequently the difference in its fluctuating velocities in the directions parallel to and normal to the direction of the drift velocity, i.e. the particle Oseen terminal velocity, is very small. This indicates that the crossing-trajectory effect on the statistics of the fluctuating velocity is negligible. For three other kinds of particles with larger time constants, the crossing-trajectory effect plays an important role and reduces the particle fluctuating velocity more in the direction normal to the direction of drift velocity than in the direction parallel to the particle drift direction; the larger the particle time constant, the greater the difference between the particle fluctuating velocity decays in the directions per-

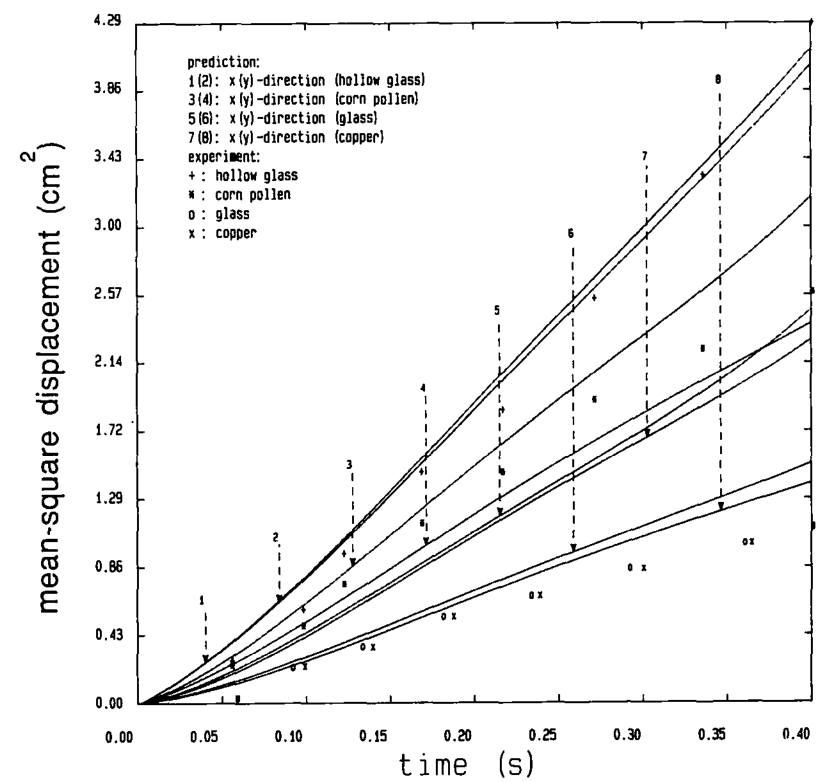

FIG. 2. Variation of particle mean-square displacements with time for four types of particles.

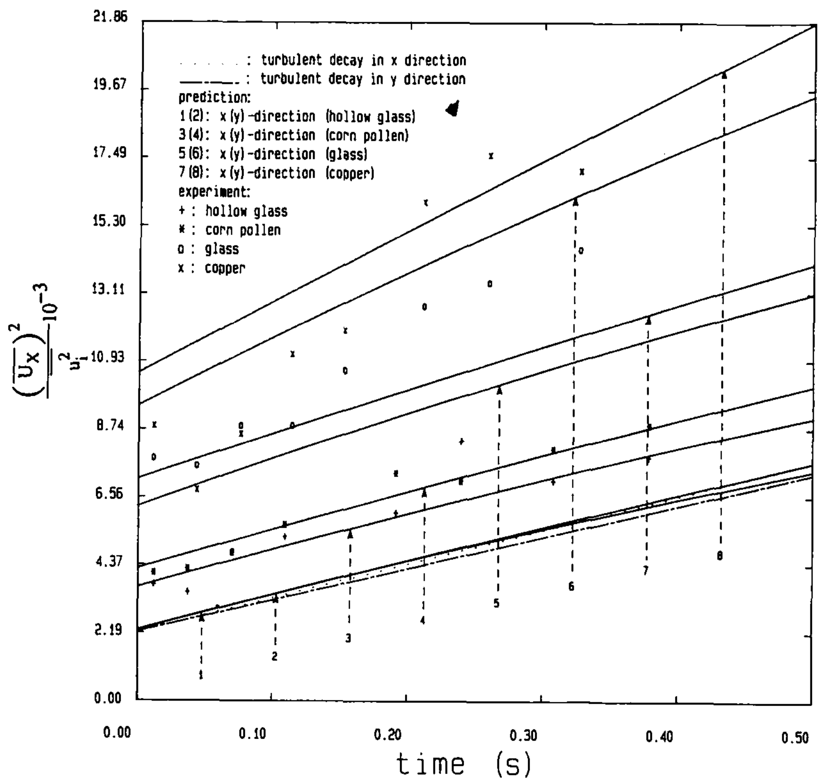

FIG. 3. Variation of particle fluctuating velocities with time.

pendicular to and parallel to the particle drift direction, a tendency consistent with the previous study of Reeks [21].

3.3. *Simulation of the experiment of Wells and Stock* [10]

Wells and Stock [10] used an identical grid system to Snyder and Lumley's to produce a turbulent air flow, but the principal direction of flow was horizontal. The mean data of the turbulent field are

$$\bar{U}_X = 655\,\mathrm{cm\,s^{-1}} \quad \bar{U}_Y = 0 \tag{22}$$

$$\overline{u_X^2} = \frac{(\bar{U}_X)^2}{54.88(X/M-7.987)} \tag{23}$$

$$\overline{u_Y^2} = \frac{(\bar{U}_X)^2}{54.88(X/M-7.987)} \tag{24}$$

$$\varepsilon = \frac{(\bar{U}_X)^3}{2M}\left[\frac{3}{54.88(X/M-7.987)^2}\right]. \tag{25}$$

The aim of this experiment was to isolate the effect of crossing-trajectories on the dispersion of the particles in the turbulent flow. The particles were charged before the grid and a uniform electrical field within the test section was used to simulate the effect of several gravity fields. The measurement of the transverse dispersions of the particles began from $X/M = 20$ for two kinds of solid glass beads.

For both types of particles and for several drift velocity values, the calculated and experimental dispersions are compared in Figs. 4 and 5. As shown in the two figures, changes in the drift velocity affect significantly the 57 μm particle, while the 5 μm particle is predominantly controlled by the turbulent flow. In addition, in the absence of V_d, it appears to be that the 57 μm particle disperses more than the 5 μm particle, but this tendency is too weak to draw a firm conclusion. We somehow agree to the theoretical predictions of Reeks and Pismen and Nir [21, 22] that in the absence of V_d and for particles whose time constants are smaller than 0.1 s, the particle dispersion is only weakly influenced by its inertia.

Figures 6 and 7 represent comparisons of calculated particle fluctuating velocity decays in the longitudinal direction (X direction) with experimental data for the 5 and 57 μm particles, respectively. Also included are the predicted particle fluctuating velocity decay in the transverse direction and the turbulent flow field decay as a reference. Again, all the tendencies observed in the experiment of Snyder and Lumley appear. In addition, in the absence of V_d, the computed particle fluctuating velocity decays for the 57 μm particle in the X and Y directions are statistically equivalent. The experimental results seem to indicate a faster decay of the particle fluctuating velocity in the case of zero drift velocity than in the case of non-zero V_d. In this paper, Wells and Stock mentioned that the differences are small and that they expect no significant influence of V_d on the decay curves of particle fluctuating velocities. Our predictions only show a slight influence

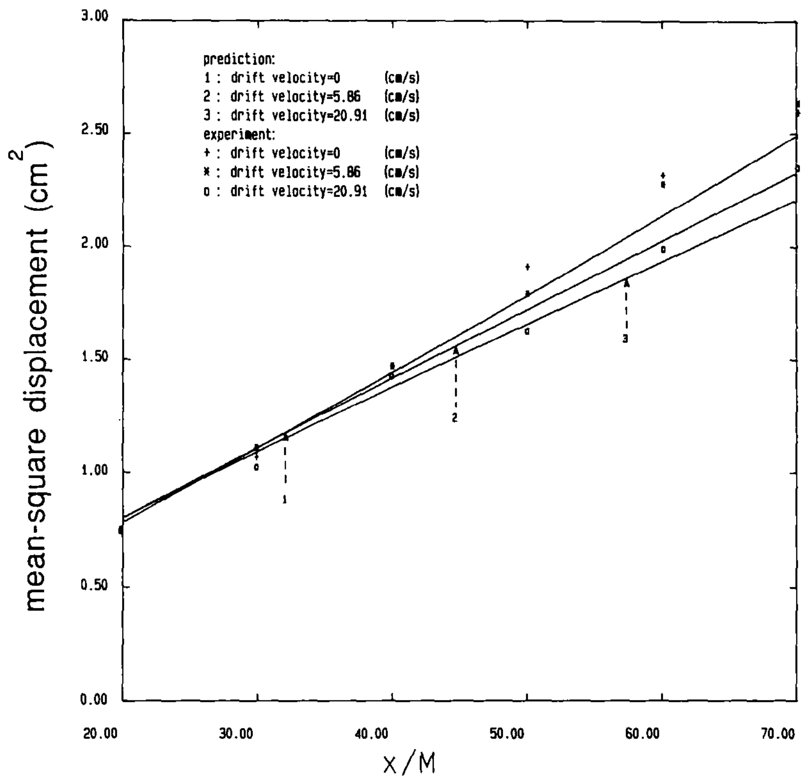

FIG. 4. Five-micron particle-transverse dispersions when V_d takes different values.

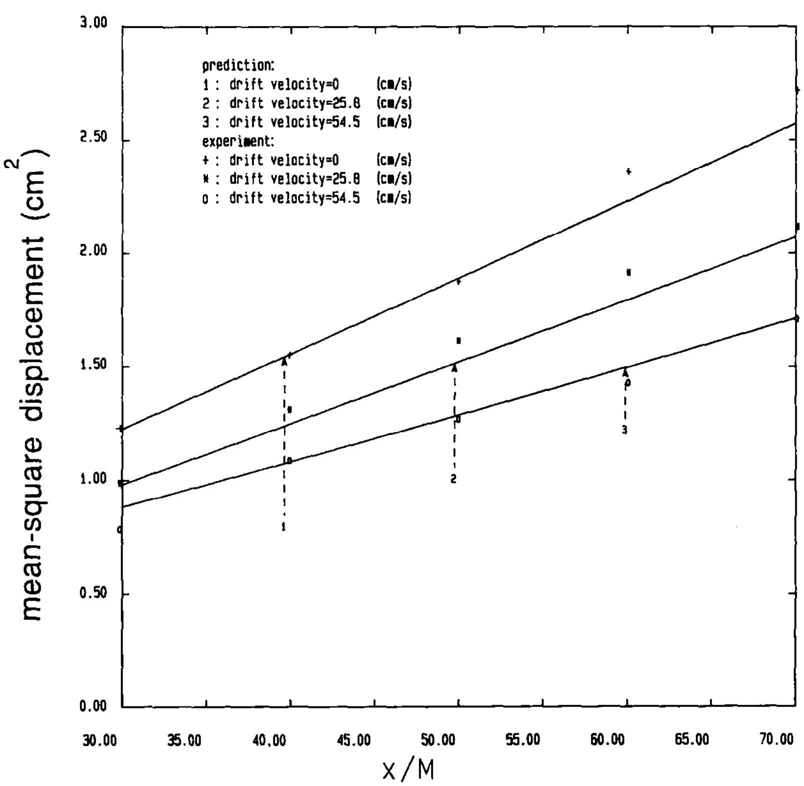

FIG. 5. Fifty-seven-micron particle-transverse dispersions when V_d takes different values.

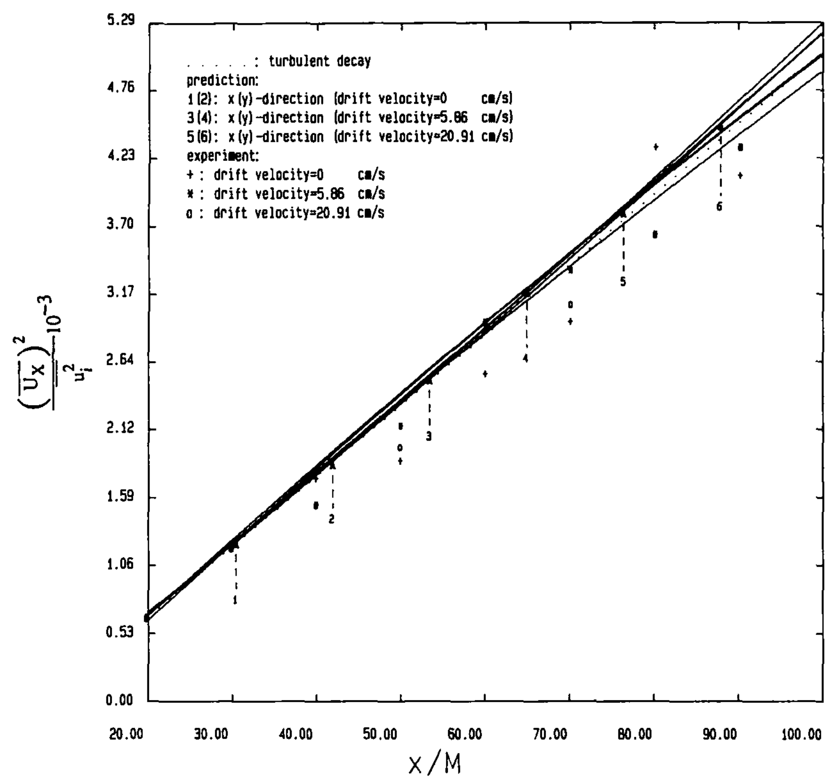

FIG. 6. Five-micron particle fluctuating velocity decay in the longitudinal and transverse directions.

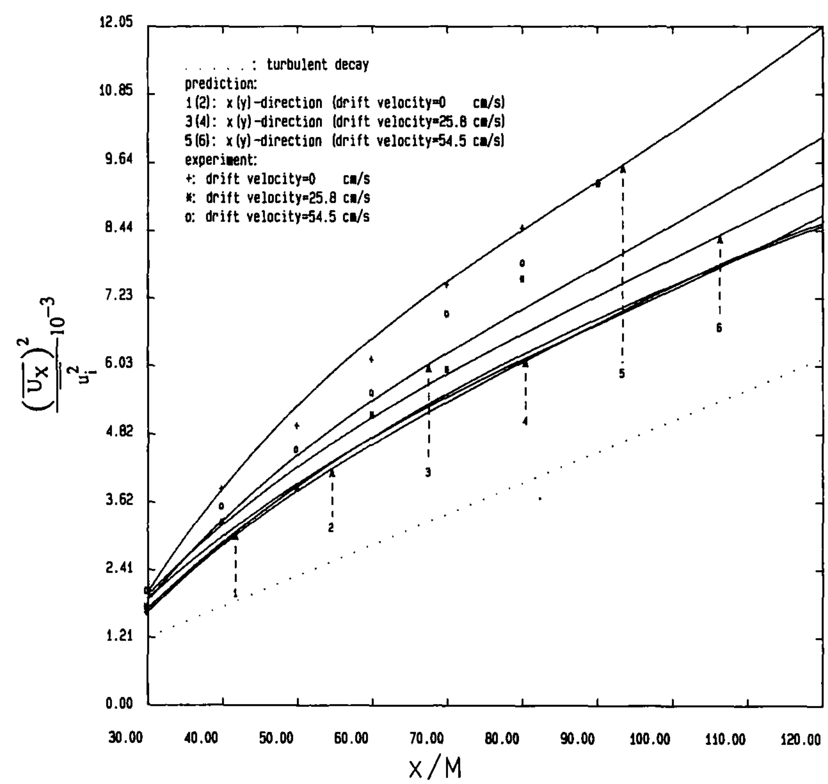

FIG. 7. Fifty-seven-micron particle fluctuating velocity decay in the longitudinal and transverse directions.

of V_d; the decay is slower in the case of zero V_d. The same tendency had been theoretically predicted by Reeks.

3.4. *Remarks*

The numerical experiments conducted by using values of Δt ranging from 2×10^{-4} to 10^{-2} s show that the computed results for the same kind of particle vary with a difference smaller than 6/100. All the predictions presented above are obtained by averaging over 5000 particles.

4. CONCLUSION

We have presented a Lagrangian model to describe the particle motion in turbulent flows. The basic idea of our model relies on the conventional time series analysis. By this method, we are able to estimate the fluid fluctuating velocity along the particle trajectory. As a validation of the present model, we have simulated numerically the two experiments of Snyder and Lumley and of Wells and Stock. On the whole, the predicted and experimental results, including particle-dispersion and particle fluctuating velocity decay, are in a fair agreement. Furthermore, the corresponding quantities for the longitudinal direction are also computed. It is shown that the crossing-trajectory effect influences unequally the particle-dispersion and particle fluctuating velocity decay in the directions normal and parallel to the gravity direction. These trends conform to the theoretical study of Reeks.

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